4.3 Notes and Examples   Name: Block: Seat:

Definition of an Integral, Integration by Geometry, Properties of Integrals

1. Warm up question 1: A rocket has velocity $v(t)$ after being launched upwards from a initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for select values of t over the interval $0 \leq t \leq 80$ seconds:

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ in (ft/sec)	5	14	22	29	35	40	44	47	49

   (a) Find the **average acceleration** of the rocket over the the time interval $0 \leq t \leq 80$

   (b) What is the meaning of $\int_{10}^{70} v(t)\, dt$ in this context?

   (c) Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\, dt$

2. Warm up question 2: Water is flowing into a tank at the rate of $r(t)$, where $r(t)$ is measured in gallons per minute, and time t is measured in minutes. The tank contains 15 gallons of water at time $t = 0$ minutes. Values of $r(t)$ for selected values of t are given in the table below.

t (minutes)	0	4	7	9
$r(t)$ in (gallons per minute)	9	6	4	3

   (a) Approximate the number of gallons of water in the tank at time $t = 9$ using a trapezoidal sum with three subintervals.

   (b) Write an integral expression of what we just estimated.

3. **Definition of Integral** (p 272) If f is defined on the closed interval $[a, b]$ and the limit of Riemann sums over partitions Δ

$$\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(c_i) \cdot \Delta x_i$$

exists, then f is said to be ________________________________on $[a, b]$ and the limit is denoted by

$$\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(c_i) \cdot \Delta x_i = \underline{\hspace{6cm}}$$

The limit is called the ________________________________. of f from a to b where the number a is called the ________________________________. and the number b is called the ________________________________.

4. **Continuity Implies Integrability** : If a function f is continuous on the closed interval $[a, b]$, then ________________________________. That is, ________________________________ exists.

5. Use the limit definition to evaluate $\displaystyle\int_{-2}^{1} 2x \, dx$.

   (a) If the base of the rectangle $\Delta x = \dfrac{b-a}{n}$, then here Δx is:

   (b) If $c_i = a + i(\Delta x)$, then here c_i is:

   (c) So the height of the rectangle $f(c_i)$ is:

   (d) $\displaystyle\lim_{\|\Delta x\| \to 0} \sum_{i=1}^{n} f(c_i) \cdot \Delta x_i = \lim_{n \to \infty} \sum_{i=1}^{n} f(c_i) \cdot \Delta x$

6. **Integration by Geometry** A definite integral describes the signed area between the function and the x-axis. The area below the x-axis is negative, and the area above the x-axis is positive.

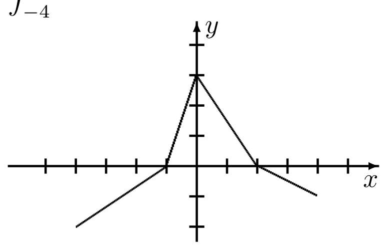
   Sketch and evaluate by using a geometric formula.

   (a) $\int_1^7 6\ dx =$

   (b) $\int_1^3 x + 2\ dx =$

   (c) $\int_{-2}^2 \sqrt{4 - x^2}\ dx =$

   (d) $\int_{-4}^4 f(x)\ dx =$



$y = f(x)$

7. **Properties of Integrals** (page 275-276)
   (a) $\int_a^a f(x)\ dx =$
   (b) $\int_b^a f(x)\ dx =$
   (c) $\int_a^b k \cdot f(x)\ dx =$
   (d) If $a < c < b$, then $\int_a^b f(x)\ dx =$
   (e) $\int_a^b f(x) + g(x)\ dx =$
   (f) If $f(x) \geq 0$ for every x in $[a, b]$, then
   (g) If $f(x) \leq g(x)$ for every x in $[a, b]$, then

8. Given $\int_0^3 f(x)\ dx = 4$ and $\int_3^7 f(x)\ dx = -1$ Find
   (a) $\int_0^7 f(x)\ dx$
   (b) $\int_3^7 2f(x)\ dx$
   (c) $\int_5^5 f(x)\ dx$
   (d) $\int_7^3 f(x)\ dx$

9. Let f and g be continuous functions. If $\int_2^6 f(x)\ dx = 5$ and $\int_6^2 g(x)\ dx = 7$, then $\int_2^6 3f(x) + g(x)\ dx =$

10. Given $\int_{10}^0 f(a)\ da = -12$ and $\int_4^{10} f(b)\ db = 7$, find $\int_0^4 f(q)\ dq$